# STOCHASTIC UPPER BOUNDS FOR AGGREGATE NETWORK TRAFFIC WITH MODELING BASED ON FRACTIONAL STABLE NOISE

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There is increasing evidence that real network traffic is well modeled using $\alpha$-stable long-range dependent stochastic processes. However, just a few studies are currently available that allow for exploiting their goodness of fit. This modeling approach can be used, for instance, to determine probabilistic upper limits for the resource demand created by real network traffic. This issue is of prime importance since this type of bounds has been found to be quite useful in diverse areas of resource management. In this paper we derive upper bound expressions for an aggregate traffic process modeled using fractional stable noise with symmetrical marginal distributions. The validity of the resulting formulas is tested with three case studies.

## 1 Introduction

A number of recent studies have shown that high burstiness and long-range dependence are important properties of several classes of traffic. Examples are Ethernet [8], WWW traffic [2] and variable-bit-rate (VBR) video traffic [1]. After years of intense research, several models have been proposed with the objective of reflecting these statistical properties of the traffic inside networks.

Some of the most relevant models presented in the past are based on fractional Brownian motion. In this context it is worth mentioning the work by Norros in [9] and by Willinger *et al.* in [12]. More recently, models like the ones introduced in [4] and [5] propose the use of an $\alpha$-stable long-range dependent stochastic process, namely fractional stable noise (FSN), to capture the statistical properties of the number of arrivals of an aggregate traffic process over time intervals. These models are a generalization of the ones presented in [9] and [12], in the sense that, rather than limiting the marginal distribution of the process to be Gaussian, $\alpha$-stable distributions are now used, which allows for a better agreement between the burstiness of the artificial process and that of real traffic.

Even though traffic models based on $\alpha$-stable distributions allow for a more accurate modeling, currently just a few studies in the context of traffic analysis and engineering design are available. It is in these areas that stochastic limits that bound the randomness of the traffic process are of prime importance. Up to the authors' knowledge no previous studies, using this novel modeling approach, have been carried out in order to provide expressions to upper bound the cumulative arrival process. In this paper we try to fill this gap by extending the concept of envelope processes to the $\alpha$-stable case.

The remaining of this paper is organized as follows. The following section introduces the notation and all the required concepts and methods. In section 3 we derive upper bound expressions for the cumulative arrival process. In Part 4 we exemplify the validity of the derived formulas through three case studies. Section 5 concludes this paper.

## 2 Background concepts and methods

For the sake of clarity, a brief overview of the involved concepts is provided in this section.

### *2.1 Symmetric α-stable random variables*

Following the notation used in [10], $S_\alpha(\sigma,\beta,\mu)$ stands for a stable distribution with index of stability $\alpha$ ($0<\alpha\leq 2$), scale parameter $\sigma$ ($\sigma\geq 0$), skewness parameter $\beta$ ($-1\leq\beta\leq 1$) and shift parameter $\mu$ ($\mu\in\Re$). In particular, a random variable $X$ is $S\alpha S$ (symmetric $\alpha$-stable) if and only if $X\sim S_\alpha(\sigma,0,0)$[1]. A useful quantity derived from the parameters of a stable distribution is its dispersion $\gamma$, defined as:

$$\gamma \overset{\Delta}{=} \sigma^\alpha \tag{1}$$

A $S\alpha S$ random variable $X$ can be defined in terms of its characteristic function, given by:

$$\Phi_x(\theta) = E[\exp(j\theta X)] = \exp(-|\sigma_x\theta|^\alpha) = \exp(-\gamma_x|\theta|^\alpha) \tag{2}$$

Let $X_1$ and $X_2$ be independent $S\alpha S$ random variables such that $X_i \sim S_\alpha(\sigma_i,0,0)$, for $i$ = 1, 2, and let $a$, $b$ be arbitrary real constants. Using their characteristic functions it can be shown that:

$$X_1 + a \sim S_\alpha(\sigma_1, 0, a) \tag{3}$$

$$a\cdot X_1 \sim S_\alpha(|a|\cdot\sigma_1, 0, 0) \tag{4}$$

$$X_1 + X_2 \sim S_\alpha(\sigma,0,0), \text{ with: } \sigma = (\sigma_1^\alpha + \sigma_2^\alpha)^{1/\alpha} \tag{5}$$

From the last two equations it can be easily shown that if a random variable $X$ is defined as $X = aX_1 + bX_2$ then $X \sim S\alpha S$ with dispersion:

[1] The notation $X \sim S_\alpha(\sigma,\beta,\mu)$ means that $X$ has the distribution $S_\alpha(\sigma,\beta,\mu)$.

$$\gamma_X = \gamma_1 |a|^\alpha + \gamma_2 |b|^\alpha \tag{6}$$

where $\gamma_1$ and $\gamma_2$ are the dispersions of $X_1$ and $X_2$, respectively.

### 2.2 *Synthesis of fractional stable noise*

A fractional stable noise (FSN) sequence $Y_j$ represents the increment during the *j-th* time interval of unit length of a self-similar process with index $H$ and stationary increments ($H$-sssi). We will consider two families of $H$-sssi processes, the linear fractional stable motion (LFSM or $L_{\alpha,H}(t)$) and the Log-fractional stable motion (Log-FSM or $\Lambda_\alpha(t)$). The LFSM process is defined as:

$$L_{\alpha,H}(t) = \int_{-\infty}^{\infty} f(t,x) \cdot M(dx) \tag{7}$$

where $M(dx)$ is an $\alpha$-stable random measure on $\Re$ with Lebesgue control measure. The *kernel* function $f(t,x)$ is given by:

$$f(t,x) = a\left[(t-x)_+^{H-1/\alpha} - (-x)_+^{H-1/\alpha}\right] + b\left[(t-x)_-^{H-1/\alpha} - (-x)_-^{H-1/\alpha}\right] \tag{8}$$

where:

$$y_+ \overset{\Delta}{=} \begin{cases} y & ; y \ge 0 \\ 0 & ; y < 0 \end{cases} \qquad y_- \overset{\Delta}{=} \begin{cases} 0 & ; y \ge 0 \\ -y & ; y < 0 \end{cases}$$

The parameters $a$ and $b$ are arbitrary real constants although some specific values and relations between them are frequently used. For instance, the process is called *balanced* when $a = b$ and *anti-balanced* if $a = -b$.

LFSM processes display long-range dependence when $H > 1/\alpha$. Since the parameter $H$ is restricted to lie within the interval (0, 1), then $\alpha$ must be greater than 1 for the long-range dependence to be present.

On the other hand, the Log-FSM process is defined as:

$$\Lambda_\alpha(t) = \int_{-\infty}^{\infty} f(t,x) \cdot M(dx) \tag{9}$$

$M(dx)$ is again an $\alpha$-stable random measure on $\Re$ with Lebesgue control measure. The *kernel* function in this case is:

$$f(t,x) = ln|t-x| - ln|x| \tag{10}$$

For the Log-FSM process, its parameters satisfy the relation $H=1/\alpha$, where $1<\alpha<2$.

From the previous definitions we can see that linear fractional stable noise (LFSN) can be obtained from LFSM as follows:

$$Y_j = L_{\alpha,H}(j+1) - L_{\alpha,H}(j) \tag{11}$$

Similarly, Log-fractional stable noise (Log-FSN) can be generated from Log-FSM by:

$$Y_j = \Lambda_\alpha(j+1) - \Lambda_\alpha(j) \tag{12}$$

We will use the term FSN to denote either LFSN or Log-FSN. It can be generalized that:

$$Y_j = \int_{-\infty}^{\infty} [f(j+1,x) - f(j,x)] \cdot M(dx) \overset{\Delta}{=} \int_{-\infty}^{\infty} g(j,x) \cdot M(dx) \tag{13}$$

This last equation can be restated as the following discrete moving-average expression (see [3] for details):

$$Y_j = \sum_{k=-\infty}^{\infty} \phi(j-k) \cdot U_k \;\; ; \; j \in \{0,1,2...\} \tag{14}$$

where $\{U_k\}$ is a set of independent and identically distributed (i.i.d.) $S\alpha S$ random variables $U_k \sim S_\alpha(1,0,0)$ and the coefficients $\phi(j\text{-}k)$ are given by:

$$\phi(j-k) = \left[ \int_k^{k+1} |g(j,x)|^\alpha \cdot dx \right]^{1/\alpha} > 0 \tag{15}$$

The dispersion of $Y_j$ can be determined applying the result stated by (6) into (14). Thus we obtain:

$$\gamma_Y = \sum_{k=-\infty}^{\infty} \phi^\alpha(k) \tag{16}$$

where we have taken into consideration that the dispersion of $U_k$ results in a unitary value. We have dropped the index since the same derivation proves that the dispersion does not depend on $j$.

It can be observed from (14) that an FSN process with parameters $H$ and $\alpha$ can be generated by a linear filter driven by symmetric $\alpha$-stable noise.

## 3 Stochastic upper bounds for aggregate network traffic

Define $W_j^*$ as the actual number of arrivals during the $j-th$ time interval of unit length in a traffic process. This discrete-time process can be modeled as a sequence $W_j$ (i.e. $W_j^* \overset{d}{=} W_j$, where $\overset{d}{=}$ means equality in distribution) defined as follows:

$$W_j = m + aY_j \;\; ; \; j \in \{0,1,2,\ldots\} \tag{17}$$

The parameter $m$ is the mean value of the number of arrivals per time unit, $a$ is a scaling factor and $Y_j$ is either a LFSN or a Log-FSN process with symmetrical marginal distributions. This traffic model was proposed and evaluated by Gallardo *et al.* in [4] for the modeling of aggregate MPEG-1 and MPEG-2 video traffic. A modified version of the same model was also proposed, in the same work [4], for several traffic classes (highly bursty Ethernet traffic, moderately bursty Ethernet traffic and WWW traffic). A similar model, intended for generic heavy traffic, was independently developed and evaluated by Karasaridis *et al.* in [5]. This one is based on an underlying LFSM process with parameters $a=1, b=0$ (cf. Equation (8)) and totally positively skewed distributions (i.e., $\alpha$-stable distributions with skewness parameter $\beta=1$). Both proposals proved to fit different levels of burstiness and correlation better than Gaussian self-similar models. In the present work we will follow the modeling approach proposed by Gallardo *et al.* in [4]. For our purposes it is only mathematically relevant that a traffic model can be expressed in the form of (17). The analyses presented here are valid for different families of FSN processes with symmetric distributions, allowing for a high degree of generality and flexibility. Accordingly, in what follows it is assumed that, in the traffic model, the constituent FSN process has symmetrical marginal distributions.

Let $A_n$ represent a cumulative arrival process with $A_0 = 0$ at the beginning of a busy period.

$$A_n = W_0 + W_1 + \ldots + W_{n-1} \quad ; \; n \in \{1,2,3,\ldots\} \tag{18}$$

Using (17) the cumulative arrival process can also be restated as:

$$A_n = nm + a(Y_0 + Y_1 + \ldots + Y_{n-1}) \quad ; \; n \in \{1,2,3,\ldots\} \tag{19}$$

Observe that the summation of samples of fractional stable noise implied by (19) becomes in fact fractional stable motion with discrete time index. Up to the authors' acknowledge, no upper-bound expressions are currently available to stochastically delimit the magnitude of a cumulative arrival process modeled with this stochastic process. Such kinds of bounds have been defined for more traditional models and have been found quite useful (e.g., see [11] section 6.2). These bounds usually consist of the mean value of the process plus some multiple of its standard deviation. However, when modeling network traffic with $\alpha$-stable stochastic processes some new analytical difficulties arise. The most evident limitation is that the value of their index of stability $\alpha$ conditions the existence of some moments. Generally speaking, they lack of second order moments, consequently the standard deviation cannot be obtained and therefore, alternative statistical descriptors for the variability of the process must be considered.

In order to provide an upper bound for $A_n$ we propose an envelope process $\hat{A}_n$ defined as:

$$\hat{A}_n = m_A(n) + K\sigma_A(n) \tag{20}$$

where $m_A(n)$ and $\sigma_A(n)$ are the instantaneous values of the mean and scale parameter of the cumulative arrival process $A_n$. $K$ is a parameter that determines the probability that the process will surpass its envelope. In order to determine the parameters of the envelope process it is useful to apply (14) and transform (19) into:

$$A_n = nm + \sum_{k=-\infty}^{\infty} U_k \left[ a \sum_{j=0}^{n-1} \phi(j-k) \right] \quad ; \; n \in \{1,2,3,\ldots\} \tag{21}$$

Now we apply expectation to (21) to obtain its mean:

$$m_A(n) = E\{A_n\} = nm + \sum_{k=-\infty}^{\infty} E\{U_k\} \left[ a \sum_{j=0}^{n-1} \phi(j-k) \right] = nm \tag{22}$$

In (22) we have used the fact that for the $S\alpha S$ random variables under consideration, with $1 < \alpha \le 2$, their mean equals the shift parameter (see [10] pp.19), which in turn is zero.
Using the result expressed by (6) into (21) it can be easily shown that the dispersion of the process $\gamma_A(n)$ is given by:

$$\gamma_A(n) = \sum_{k=-\infty}^{\infty} \left| a \sum_{j=0}^{n-1} \phi(j-k) \right|^{\alpha} \gamma_{U_k} \tag{23}$$

After some simple algebraic steps, and taking into consideration that the dispersion $\gamma_{U_k}$ results in a unitary value, we obtain the following expression for the scale parameter of the process $A_n$:

$$\sigma_A(n) = [\gamma_A(n)]^{1/\alpha} = |a| \cdot \left[ \sum_{k=-\infty}^{\infty} \left| \sum_{j=0}^{n-1} \phi(j-k) \right|^{\alpha} \right]^{1/\alpha} \tag{24}$$

Finally, the parameter $K$ is uniquely determined by fixing an overshoot probability $\varepsilon$ (i.e., probability that the cumulative arrival process $A_n$ will surpass its envelope process $\hat{A}_n$), as evidenced observing that:

$$P\{A_n > \hat{A}_n\} = P\{A_n > m_A(n) + K\sigma_A(n)\} = P\left\{ \frac{A_n - m_A(n)}{\sigma_A(n)} > K \right\} = \varepsilon \tag{25}$$

It is important to observe that shifting $A_n$ by $m_A(n)$ and scaling the result by $1/\sigma_A(n)$ is in fact a normalization procedure. Thus,

$$\text{if } X = \frac{A_n - m_A(n)}{\sigma_A(n)} \quad \text{then } X \sim S_\alpha(1,0,0) \tag{26}$$

Unfortunately, in general there are no closed form expressions for the distribution of an $\alpha$-stable random variable that would allow us to determine $K$ analytically from the probability $\varepsilon$. Nevertheless, the characteristic function of $X$ can be numerically inverted to obtain its density and then a table of stored values of $K$ versus $\varepsilon$ (i.e., $K$ versus $P\{X > K\}$) can be generated.

It may be useful to analyze a long trace of input traffic by splitting it into smaller subsequences of fixed length. In this way, the number of parameters of the envelope process to be computed can be known in advance instead of being randomly determined by the duration of busy periods. Assume that we observe a traffic process using a window of size $N$ and that the cumulative arrival process is reset at the beginning of each window (i.e., $A_{Ni} = 0$ where $i \in \{0,1,2,\ldots\}$, with the beginning of the next window coinciding with the end of the current one). Now let $n$ denote the position of the current sample of the cumulative arrival process relative to the beginning of the current window as follows:

$$A_{Ni+n} = W_{Ni} + W_{Ni+1} + \ldots + W_{Ni+n-1} \quad ; \; n \in \{1, \ldots, N\} \tag{27}$$

In terms of the new parameters, the envelope for this process is:

$$\hat{A}_{Ni+n} = m_A(Ni+n) + K\sigma_A(Ni+n) \tag{28}$$

Similar analyses to the ones that led to (22) and (24) can be used to show that the instantaneous mean $m_A(Ni+n)$ and scale parameter $\sigma_A(Ni+n)$ of the windowed cumulative arrival process $A_{Ni+n}$ are given by:

$$m_A(Ni+n) = nm \tag{29}$$

$$\sigma_A(Ni+n) = |a| \cdot \left[ \sum_{k=-\infty}^{\infty} \left| \sum_{j=0}^{n-1} \phi(j-k) \right|^{\alpha} \right]^{1/\alpha} \tag{30}$$

The parameter $K$ would be obtained as previously explained. It can be observed that the window length has no effect in the bounding function of the envelope process.

## 4 Numerical examples

The validity of the previously derived expressions was evaluated using three traces of aggregate network traffic. In the first two cases we generated traces of artificial traffic as explained in section 2.2. In these two cases our purpose was to exemplify the use of our generalized envelope processes with both Gaussian and stable non-Gaussian models. In the third case we used an aggregate MPEG-1 traffic trace in order to provide a more detailed example.

### *4.1 Tests with artificial traces*

In the first case a synthetic trace of self-similar traffic consisting of 10,000 samples was generated using fractional Gaussian noise (fractional stable noise with the parameter $\alpha$ set to 2). The Hurst parameter $H$ was set to 0.80 so that it matched typical measured values for some types of network traffic (e.g., see [4]). With this information ($\alpha$=2, $H$=0.80) the time-dependent parameters of the corresponding envelope process were computed for window sizes ranging from 1 to 50. Tables for normalized Gaussian distributions helped to quickly determine the value of $K$ that satisfied an overshoot probability $\varepsilon$. Table 1 shows the predicted overshoot probabilities $\varepsilon$ and Table 2 shows the simulation results (sample mean values). It is worth mentioning that the simulation results follow a distribution $S_2(1,0,0)$, which equals a non-normalized Gaussian distribution of zero mean and standard deviation $\sqrt{2}$ so that in order to compare the results against values from a normalized distribution an appropriate scaling by $1/\sqrt{2}$ was required.

**Table 1.** Gaussian case: predicted overshoot probability $\varepsilon$ (obtained from a table of $N(0,1)$).

| $K$ | 1 | 2 | 3 | 5 |
|---|---|---|---|---|
| $\varepsilon$ | 0.159 | 0.0228 | 0.00135 | 2.87E-7 |

**Table 2.** Gaussian case: experimentally observed overshoot probability $\varepsilon$.

| Window size | $K$ | | | |
|---|---|---|---|---|
| | 1 | 2 | 3 | 5 |
| 1 | 0.1544 | 0.0176 | 0.0002 | 0.0000 |
| 2 | 0.1530 | 0.0178 | 0.0001 | 0.0000 |
| 3 | 0.1525 | 0.0184 | 0.0001 | 0.0000 |
| 5 | 0.1524 | 0.0183 | 0.0000 | 0.0000 |
| 10 | 0.1504 | 0.0204 | 0.0000 | 0.0000 |
| 20 | 0.1520 | 0.0205 | 0.0000 | 0.0000 |
| 30 | 0.1486 | 0.0160 | 0.0000 | 0.0000 |
| 50 | 0.1427 | 0.0233 | 0.0000 | 0.0000 |

In the second case, an $\alpha$-stable non-Gaussian model was used to generate a trace consisting of 10,000 samples. The model was based on balanced fractional stable noise with parameters $\alpha = 1.95$ and $H$=0.90. We considered the same window sizes as in the previous example. The value of $K$ that satisfies an appropriate probability $\varepsilon$ had to be determined from a tailored table of $K$ versus $\varepsilon$ (i.e, $K$ versus $P\{X > K\}$ where $X \sim S_{1.95}(1,0,0)$). In order to generate such table, the characteristic function was numerically inverted to obtain a punctual representation of the probability density function. Table 3 shows the predicted overshoot probabilities $\varepsilon$ and Table 4 shows the simulation results.

**Table 3.** Non-Gaussian case: predicted overshoot probability $\varepsilon$ from the distribution $S_{1.95}(1,0,0)$.

| $K$ | 1 | 2 | 3 | 5 |
|---|---|---|---|---|
| $\varepsilon$ | 0.2401 | 0.0808 | 0.0199 | 0.0016 |

**Table 4.** Non-Gaussian case: experimentally observed overshoot probability $\varepsilon$.

| Window size | $K$ | | | |
|---|---|---|---|---|
| | 1 | 2 | 3 | 5 |
| 1 | 0.2378 | 0.0704 | 0.0124 | 0.0006 |
| 2 | 0.2385 | 0.0707 | 0.0130 | 0.0007 |
| 3 | 0.2371 | 0.0700 | 0.0127 | 0.0007 |
| 5 | 0.2377 | 0.0701 | 0.0129 | 0.0008 |
| 10 | 0.2374 | 0.0725 | 0.0117 | 0.0009 |
| 20 | 0.2335 | 0.0704 | 0.0112 | 0.0010 |
| 30 | 0.2369 | 0.0692 | 0.0129 | 0.0011 |
| 50 | 0.2311 | 0.0666 | 0.0146 | 0.0000 |

It can be observed that, in both cases, the proposed envelope process closely bounds the behavior of the synthetic traffic trace. The experimentally obtained values for the overshoot probabilities were, in nearly all the cases, upper bounded by the predicted ones. It is clear, as previously stated, that the window size has a minimal effect in the results. Notice that we can upper bound individual samples of the arrival process by setting the window size to a unitary value. It is also worth mentioning that since the number of samples was limited to 10,000 the minimum resolution in the observed overshoot probability is 0.0001.

### 4.2 *Test with a real aggregate video trace*

In our last case study we applied the proposed envelope process to a real aggregate stream. This traffic flow came from the aggregation of 16 MPEG-1 video sequences obtained from the University of Würzburg in Germany. The resulting sequence $W_j^*$ consisted of 2627 samples. Following [4] this traffic sequence was modeled as indicated by (17) and using balanced fractional stable noise with parameters $\alpha$=1.95 and $H$=0.90. The model parameters $m$ and $a$ were determined as follows. The parameter $m$ was computed as the sample mean of the aggregate sequence, yielding a value of $m = 3655758$[bits/time_unit]. The scaling factor $a$ was found as the ratio of the scale parameter of the sequence $(W_j^* - m)$ to the scale parameter of $Y_j$ (the estimation method for the former was the sample characteristic function, similar to the one in [6] and [7], and the latter was computed from (16) and (1)). This scaling factor was found to be $a = 256859.4$ .

The envelope processes we considered for this set of tests were computed as to satisfy the same overshoot probabilities as in the previous set of simulations. Since the traffic class under consideration uses the same underlying fractional stable noise process as in the previous case study, we used the data from Table 3 to help us to determine the parameter $K$ (cf. Equation (28)). For comparison purposes we reproduce this information in one row of Table 5.

The time-dependent elements of the envelope process were computed considering a window of size one. The resulting overshoot probabilities from these tests are shown in the last row of Table 5. It is observed that the proposed envelope indeed was able to upper bound the real aggregate traffic trace. Finally, Figure 1 shows the traffic trace along with the envelope processes.

**Table 5.** Aggregate MPEG-1 traffic: overshoot probability $\varepsilon$ .

| $K$ | 1 | 2 | 3 | 5 |
|---|---|---|---|---|
| $\varepsilon$ | 0.2401 | 0.0808 | 0.0199 | 0.0016 |
| $\varepsilon$ (experimental) | 0.2098 | 0.0716 | 0.0187 | 0.0011 |

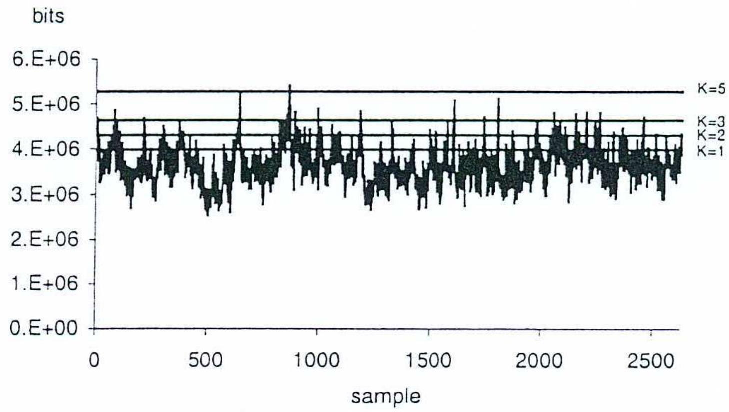


**Figure 1.** Aggregate MPEG-1 traffic and envelope processes.

## 5 Conclusion

Current trends in traffic modeling suggest that the best representation of aggregate traffic in networks is achieved using $\alpha$-stable long-range dependent stochastic processes. One of the strongest qualities of this modeling approach resides in its parsimonious representation of reality. However, its current applicability is limited by the increased computational cost, limited mathematical tractability and scarce availability of performance analyses. The research effort reported in this work addresses this last issue. We used a traffic model based on fractional stable noise to determine stochastic upper bounds for the corresponding cumulative arrival process. Our proposed bounds can be considered as an extension of the concept of envelope processes to the $\alpha$-stable case, which includes fractional Brownian motion as a particular case. The simulation results show that the obtained expressions are indeed able to stochastically limit the cumulative arrival sequence. These bounds have very promising applications in several areas of resource management including admission control and congestion control.

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